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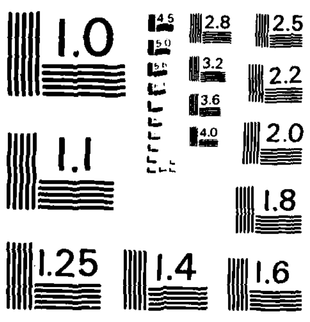

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ANALYSIS OF THREE-DIMENSIONAL VISCOUS INTERNAL FLOWS

K.N. GHIA

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U. GHIA

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ANALYSIS OF THREE-DIMENSIONAL VISCOUS INTERNAL FLOWS

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Applied Mechanics
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This research was supported by the Air Force Office of Scientific Research, Bolling Air Force Base, under AFOSR Grant No. 80-0160 with Dr. James D. Wilson as Technical Monitor.

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Flow Separation, Secondary Flows
k-ε Turbulence Model, Low-Re Modelling
Semi-Implicit & Implicit Numerical Methods
Multi-Grid & Strongly Implicit Techniques
ADI; Block Gaussian Elimination Method

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report describes the technical progress achieved in the research sponsored by the Air Force Office of Scientific Research during the period between March 1982 and February 1983. The research pursued has been conveniently grouped into three general categories. In the first category itself, two different areas were studied. ~~These~~ are (i) analysis of laminar duct flows, and ~~(ii)~~ study of laminar and turbulent separated flows. These studies were aimed at acquiring a better understanding of isolated physical phenomena

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significant to turbomachinery applications via the use of appropriate model problems. The second category ~~of~~ research is aimed at obtaining flow-dependent computational grids efficiently, so that critical regions can be accurately modeled. The final category ~~of~~ research ~~pursued~~ consisted of the analysis of numerical methods, with the goal of improving the efficiency and accuracy of the various methods developed and implemented. In the first category of research, preliminary fine-grid marching solutions were obtained in the entrance region of the duct for eight different duct configurations. The subject of streamwise separation was examined, using the model problem of laminar flow through a constricted asymmetric channel. True transient results were obtained for several flow configurations with extremely fine grids, so as to provide benchmark solutions which can permit assessment of other solutions obtained using approximate methods. Turbulence modelling was pursued, with the wall region being described by the low-Re modelling. Although the wall region can be modeled more accurately by this method, the fine grids it required retards the convergence rate of the approximate factorization method used. For the second category of the present research, flow-dependent grids were generated for a 1-D nonlinear viscous Burgers' equation. For the first time, accurate results were computed using totally central-difference schemes for Re up to 10^4. Finally, in the last category, in the area of semi-implicit methods, a multi-grid method was developed to provide fine-grid solutions for the Neumann problem. In the area of implicit numerical techniques, the fully implicit method, which uses block Gaussian elimination technique to solve the Poisson problem, was implemented in the solution of the unsteady Navier-Stokes equations; it provided accurate 2-D transient solutions which permitted the assessment of three-dimensionality in the experimental results for the model problem investigated.

ABSTRACT

This report describes the technical progress achieved in the research sponsored by the Air Force Office of Scientific Research during the period between March 1982 and February 1983. The research pursued has been conveniently grouped into three general categories. In the first category itself, two different areas were studied. These are (i) analysis of laminar duct flows, and (ii) study of laminar and turbulent separated flows. These studies were aimed at acquiring a better understanding of isolated physical phenomena significant to turbomachinery applications via the use of appropriate model problems. The second category of research is aimed at obtaining flow dependent computational grids efficiently, so that critical regions can be accurately modeled. The final category of research pursued consisted of the analysis of numerical methods, with the goal of improving the efficiency and accuracy of the various methods developed and implemented. In the first category of research, preliminary fine-grid marching solutions were obtained in the entrance region of the duct for eight different duct configurations. The subject of streamwise separation was examined, using the model problem of laminar flow through a constricted asymmetric channel. True transient results were obtained for several flow configurations with extremely fine grids, so as to provide bench-mark solutions which can permit assessment of other solutions obtained using approximate methods. Turbulence modelling was pursued, with the wall region being described by the low-Re

modelling. Although the wall region can be modeled more accurately by this method, the fine grids it requires retards the convergence rate of the approximate factorization method used. For the second category of the present research, flow-dependent grids were generated for a 1-D nonlinear viscous Burgers' equation. For the first time, accurate results were computed using totally central-difference schemes for Re up to 10^4. Finally, in the last category, in the area of semi-implicit methods, a multi-grid method was developed to provide fine-grid solutions for the Neumann problem. In the area of implicit numerical techniques, the fully implicit method, which uses block Gaussian elimination technique to solve the Poisson problem, was implemented in the solution of the unsteady Navier-Stokes equations; it provided accurate 2-D transient solutions which permitted the assessment of three-dimensionality in the experimental results for the model problem investigated.

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SECTION 1

OBJECTIVES

The objective of the present study was to develop analysis and improved understanding of viscous internal flows for a class of complex three-dimensional configurations related to turbomachinery, using appropriate model problems.

(1) Laminar incompressible flow with streamwise separation was to be studied with the help of the model problem of a doubly infinite channel with an asymmetric constriction. In particular, accurate and efficient numerical solutions were to be obtained for high-Reynolds number separated flows, where the convergence behavior of many of the existing numerical schemes becomes unsatisfactory. Also, 1-D stretching transformations were to be examined to improve the grid and, hence, the behavior of the numerical solution near the inflow and outflow boundaries. Finally, with the aid of this model problem, the flow structure was to be scrutinized to determine whether, or not, a series of like-rotating vortex structures, characteristic of unsteady flow, can exist within the boundary layer for steady flow. Furthermore, the study of turbulent separated flow was to be carried out to understand the low-Reynolds number turbulence modelling techniques required near zero-slip boundaries in the second-order turbulence closure of the time-averaged Navier-Stokes equations using the (k,ε) equations.

(2) For complex viscous flows exhibiting multiple regions of high gradients, an adaptive grid was to be determined by numerical coordinate transformation via Poisson equations, with source terms

related to the flow and its gradients. The 1-D nonlinear viscous Burgers' equation was to be used as the model problem for determining the flow-dependent grid. The driving force for controlling the grid-point motion was to be formulated such that it can be related to the source term in the Poisson equations for numerical coordinate transformation.

(3) The efficiency and accuracy of semi-implicit and implicit methods were to be investigated using model problems. In the first category, the multi-grid (MG) method was investigated and the various associated operators, namely, restriction, prolongation, smoothing and coarse-grid correction-equation, were to be determined so as to enhance the overall convergence of the MG iteration process. The effect of using the full multi-grid (FMG) algorithm versus the cycling multi-grid algorithm were to be studied, particularly for high-Re solutions obtained by 'continuation'. In the latter category, the analysis using a fully implicit method was to be improved for the solution of two-dimensional unsteady incompressible Navier-Stokes equations using orthogonal curvilinear coordinates. This study of the fully implicit method was to be carried out using the model problem of flow in a doubly infinite backstep channel for which reliable experimental data are available.

The research performed in each of these areas is described briefly in the next section; the main results and conclusions obtained are also summarized.

SECTION 2

DESCRIPTION OF SIGNIFICANT ACCOMPLISHMENTS

All three areas of research proposed were initiated and the specific achievements made in these studies during the reporting period are briefly described in the following sub-sections.

2.1 Duct Flows, Streamwise Separation and Turbulence Modelling

The three-dimensional incompressible flow inside curved ducts of regular cross sections was formulated earlier using the parabolized Navier-Stokes equations in terms of primitive variables. This formulation follows the analysis of U. Ghia, K. Ghia and Goyal (1979) and requires the solution of Neumann boundary-value problems for the determination of pressure and a correction-velocity scalar potential. Preliminary results using very fine grids have now been obtained during this reporting period. Such fine grids are necessary for accurate resolution of the critical regions in the flow field. The streamwise variation of centerline axial velocity is depicted in Fig. 1 for four different duct configurations. The duct configurations are denoted by a sequence of three letters. The first letter indicates whether the duct is straight (S) or curved (C). The second letter indicates whether the cross section is square (S) or polar (P). The last letter corresponds to uniform grids (U) or stretched grids (S). For the straight duct configurations, the centerline velocity increases monotonically and asymptotes to a value of approximately 2.1. In the case of curved ducts, the centerline velocity follows the curve corresponding to straight ducts up to a certain streamwise distance,

after which the curvature effect predominates, resulting in a drop of the centerline velocity. Further downstream, the interaction between the viscous and the centrifugal effects leads to a damped oscillatory approach of the centerline velocity to its asymptotic value. Additional results are being obtained and will be documented in the next reporting period.

Laminar incompressible flow with streamwise separation was studied further with the help of the model problem of a doubly infinite channel with an asymmetric constriction. Unsteady incompressible Navier-Stokes equations in a general orthogonal curvilinear coordinate system were solved using the direct Block Gaussian Elimination (BGE) method for the stream-function equation, together with an ADI method for the vorticity-transport equation. The conservation form of the vorticity-transport equation has been used, with the convective transport derivatives being approximated by central differences. Transient flow solutions for the doubly infinite channel have been obtained for Reynolds number up to 1000, using several computational grids, the finest one being (135, 33). Typical transient results for Reynolds number Re = 1000 for a channel configuration are shown in Fig. 2 at a characteristic time of T = 8. Here, time has been nondimensionalized by the ratio of the inlet channel height to the mean inflow velocity. As seen in Fig. 2, a large separated-flow region of a rather complex nature is formed along the lower wall. It contains two discrete developed eddy structures, both rotating in a clockwise sense. This transient series of vortices formed

on the lee side of the channel constriction shows qualitative resemblance with an unstable series of like-rotating vortex structures in the separating boundary layer near the rear stagnation point on a blunt body shown by Prandtl and Tietjens (1934). Figure 2 also shows a small separated flow region existing along the upper channel wall. The final steady-state results for this flow configuration are presented in Figs. 3-5. The earlier transient structure of the series of co-rotating eddies has now coalesced into a single recirculating eddy, characteristic of steady flow, just downstream of the asymmetric constriction as shown in Fig. 3. The appropriate vorticity contours for this configuration are shown in Fig. 4. Earlier calculations made for the same problem with an inappropriate grid distribution had led to completely incorrect 'steady-state' results for this problem. Finally, the transverse profiles of the total velocity vector are shown at selected streamwise locations in the channel in Fig. 5. The present results were compared with the work of Smith (1976), who has extensively examined and applied Stewartson's triple-deck asymptotic analysis to flow in pipes and channels with symmetric and asymmetric constrictions or dilations. The present results agree satisfactorily with the theoretical predictions of Smith (1976). A paper based on these results by Osswald and K. Ghia (1983) is to be published in the Journal of Computational Physics.

Turbulent separated flow was studied using second-order closure via the $(k-\varepsilon)$ two-equation model. The wall region was treated using the low-Reynolds number model. The model problem of flow

over a class of general two-dimensional bodies was considered. Of particular interest was the flow configuration studied experimentally by Ota and his co-workers (1976, 1978). Figure 6 shows the distribution of the turbulent kinetic energy obtained from the present analysis for a slightly blunt-shouldered body and compared with the measurements of Ota and Narita (1978) for a completely sharp-shouldered thick plate. As expected, the computed separation was milder than the measured one. When the streamwise dimension of the predicted separation bubble was scaled up to match with the measured separation bubble, the computed results within the separated-flow region compared well with the corresponding measured data as shown in Fig. 6. A paper based on this work by Abdelhalim, U. Ghia and K. Ghia (1983) is to be published in Journal of Fluids Engineering.

2.2 Numerical Grid Generation

Using the model problem of asymptotic duct flow, K. Ghia, U. Ghia and Shin (1983a) have shown that important physical processes often occur in very thin regions which, may or may not lie adjacent to boundaries. In order to obtain an accurate and economical simulation of these thin shear layers, whose locations are not known apriori, formulation of an adaptive grid was undertaken. Initially, a one-dimensional unsteady heat-transfer problem was studied. The criterion used for grid-point movement was that, at each discrete time level, the fraction of the total number of grid points placed inside a region be proportional to the fraction of the total temperature variation occurring over that region.

Although the preliminary results were encouraging, this approach was shelved temporarily in lieu of the analysis developed more recently by the present researchers.

In the simulation of viscous flows at high-Reynolds number, the nonlinear convective terms become large in comparison with the viscous terms. In this circumstance, it becomes difficult to obtain a wiggle-free solution using a central-difference discretization for the convective terms. A new adaptive grid generation analysis was developed and applied to the viscous nonlinear Burgers' equation. The grid adaption criterion consisted of minimizing the coefficient of the nonlinear convective term. As shown in Fig. 7 for the flow-dependent adaptive grid, the desired number of grid points have migrated to the region of the high gradient so as to limit the magnitude of truncation errors. To the authors' knowledge, this is the first analysis that yields smooth solutions for Reynolds number $Re = 10^4$, using a fully central-difference scheme. The wiggle-free computed results are in excellent agreement with the exact results, as shown in this figure. Additional results obtained are documented in a paper for presentation at the ASME Spring Conference and will be reported in the next reporting period.

2.3 Numerical Methods

The velocity-pressure formulation of the Navier-Stokes equations for confined viscous flows generally requires the solution of a Neumann boundary-value problem for the determination of pressure [e.g., K. Ghia et al. (1979)]. Therefore, a flow problem which

requires solution of a Neumann Poisson problem was studied using some implicit iterative schemes. In contrast to Dirichlet Poisson problem, the coefficient matrix of the vector-matrix equation for a Neumann problem is singular. All but one of the scalar difference equations are linearly independent, leading to one zero eigenvalue for the system and the solution of these algebraic equations contains an undetermined arbitrary additive constant. Furthermore, for the solution of the Neumann problem, an integral constraint needs to be satisfied. This constraint requires that the area integral of the source function be exactly equal to the line integral of the boundary-value distribution. Any discrepancy in satisfying this constraint requires that the source term be modified uniformly such that, with the modified source term, the integral constraint is satisfied exactly. From the multigrid analysis which has been developed for this problem, it was concluded that the modification of the source term should be permitted only on the finest grid. This procedure is formulated so as to lead automatically to satisfaction of the integral constraint on the coarser grids. For example, the form of the restriction operator to be used is found to be an averaging operator with certain weights associated with the fine-grid mesh points. Thus, in the interior of the computational domain, a nine-point averaging operator is needed whereas, on the boundary, it is a six-point averaging operator and, finally, for a corner, it is a four-point averaging operator.

The use of the appropriate operators made it possible to obtain converged solutions using the multi-grid procedure and carry out a comparative study for a model problem. Typical results are presented in Fig. 8 showing the effect of using various smoothing operators in the multigrid method, with four grid levels and the finest grid of (41 x 41) points. With Gauss-Seidel (GS) iteration as the smoothing operator, the convergence rate of the 4-grid MG procedure was approximately 4 times faster than a single-grid calculation on a (41 x 41) grid using optimized SOR iterations. With the alternating direction implicit (ADI) scheme as the smoothing operator, the convergence rate of the MG procedure was very similar to that using GS. However, the overall MG-ADI scheme showed a sensitivity to the step size Δt used in the ADI iterations. Finally, the strongly implicit (SI) procedure was used as the smoothing operator, and the convergence rate of the MG-SI procedure was about an order of magnitude faster than that for the single-grid calculations using SI iterations. The MG-SI scheme was also insensitive to the step-size Δt in the difference equations. It should also be mentioned that the computational gain increases with increase in mesh refinement for the finest grid. Detailed results were obtained and a paper based on this work by U. Ghia, K. Ghia and Ramamurti (1983) was presented at the Aerospace Sciences Meeting held at Reno, Nevada, in January 1983; this paper is submitted for publication in AIAA Journal.

A direct method for the solution of 2-D unsteady incompressible flow was developed by Osswald and Ghia (1981) such that transport equation is solved using an ADI scheme, whereas a Block Gaussian

Elimination (BGE) scheme is used for the solution of the Dirichlet Poisson problem in the derived variables of vorticity and stream function. The efficiency and accuracy of this method was studied using a model Dirichlet problem. Typical results of this study are presented in Tables 1-3, where the results of the semi-direct (SD) method of Martin (1978) are also included.

Tables 1 and 2 present the results of an efficiency study. As seen from Table 1, the direct BGE procedure proved to be more efficient than the iterative SD method in all the cases tested. As the stretching ratio (SR) increases, a marked degradation occurs in the efficiency of the SD technique. With a high stretching ratio and increased number of grid points, the SD method failed to converge. A comparison of these two techniques on the basis of accuracy is shown in Table 3.

The method developed by Osswald and Ghia (1981) was refined by incorporating in it improvements which could lead to true-time simulation of physical problems and increase the accuracy and efficiency of the overall solutions. To this end, a clustered grid distribution was arrived at, such that all of the local length scales of the flow problem are honored. Further, consistent inflow and outflow boundary conditions were derived using an asymptotic analysis. Finally, separated flow inside a backstep channel was selected as a model problem, since reliable experimental data are available for this problem. The computed results in the entire laminar range were compared extensively with the experimental data of Armaly and Durst (1980) as well as with the limited data of

Denham and Patrick (1974). These comparisons, as well as other detailed results, were documented in an invited paper by K. Ghia, Osswald and U. Ghia (1983b), presented at Long Beach, California, January 1983. The transient results for a flow configuration with Reynolds number Re_D = 1200 based on hydraulic diameter, are shown in Figs. 9-10. The streamline contours of this transient flow, as it reaches steady state, are shown in Figs. 9a-d, whereas Figs. 10a-d show tne corresponding vorticity contours. The strong adverse pressure gradient resulting from the sudden expansion near the backstep also causes separation on the upper wall. To the authors' knowledge, these results are first of their kind and serve to establish credibility in the analysis developed. A paper describing the analysis, as well as a complete set of results in the laminar range, is submitted to Journal of Fluid Mechanics.

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SECTION 3

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Ghia, K.N., Ghia, U. and Shin, C.T. (1983a), "Study of Asymptotic Incompressible Flow in Curved Ducts Using a Multi-Grid Technique," to appear in Journal of Fluids Engineering.

Ghia, K.N., Osswald, G.A. and Ghia, U. (1983b), "Study of Incompressible Viscous Flow in a Backstep Channel Using a Fully Implicit Method," submitted for publication in Journal of Fluid Mechanics.

Ghia, U., Ghia, K.N. and Ramamurti, R. (1983), "Multi-Grid Solution of Neumann Pressure Problem for Viscous Flows Using Primitive Variables," submitted for publication in AIAA Journal.

Ghia, U. and Abdelhalim, A. (1983), "Longitudinal Flow Along Circular Cylinders and Thick Plates, Including Blunt Leading-Edge Separation," to appear in AIAA Journal.

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SECTION 4

PROFESSIONAL PERSONNEL

The principal investigators for the research reported herein were Professors K.N. Ghia and U. Ghia, of the Department of Aerospace Engineering and Applied Mechanics, University of Cincinnati. They were assisted, periodically, by Mr. G.A. Osswald and Mr. C.T. Shin, graduate students pursuing their advanced degrees in the same Department. Dr. A.A. Abdelhalim, formerly graduate student in the Aerospace Engineering and Applied Mechanics Department, contributed by aiding in the preparation and presentation of a technical paper based on his Ph.D. dissertation completed earlier.

SECTION 5

SCIENTIFIC INTERACTIONS - SEMINAR AND PAPER PRESENTATIONS

Invited Lectures and Papers

Ghia, K. (1983), "A Time Dependent-Implicit Technique for the Solution of Incompressible Navier-Stokes Equations," ICASE, NASA Langley Research Center, Hampton, Virginia, February 1983.

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Ghia, U., Ramamurti, R. and Ghia, K.N. (1982), "A Multigrid Method for the Neumann Poisson Problem in Curvilinear Orthogonal Coordinates," presented at the AIAA 8th Annual Mini-Symposium on Aerospace Science and Technology, AFIT, WPAFB, Ohio.

Turner, M.G., Ghia, K.N., Ghia, U. and Keith, J.S. (1982), "Analysis of Biquartic Splines for Higher-Order Accurate Solutions of Differential Equations," presented at the AIAA 8th Annual Mini-Symposium on Aerospace Science and Technology, AFIT, WPAFB, Ohio.

SECTION 6

TECHNICAL APPLICATIONS

Of the various areas of research pursued, two appear to be most useful to the technical community. The multi-grid solution procedure formulated for the Neumann boundary-value problem is a unique capability developed in the present research. The conclusions drawn from this research are far-reaching as they distinctly demonstrate the significant role of the various operators in the overall efficiency of the multi-grid method and are, hence, very useful in the application of the method to flow problems in general. Secondly, the unsteady flow solution procedure using time marching and block-Gaussian elimination yields useful information about transient separated internal flows. Both of these developments provide highly accurate benchmark solutions for the problems to which these have been applied so far. The unsteady flow computational program is presently being adapted, by General Electric Company personnel, to the determination of flow through an orifice plate mounted axisymmetrically in a pipe.

Some of the research completed earlier under the sponsorship of the Air Force has recently received further interest. A procedure has been developed earlier for the solution of the incompressible Navier-Stokes equations in primitive variables, with pressure determined from a Neumann boundary-value problem. The details of the analysis and a listing of the associated computer program were requested by, and supplied to, Dr. Julius Harris

of the NASA-Langley Research Center. Also, the technical personnel (Drs. Delaney and Rhie) at Detroit Diesel-Allison have been regularly interacting with the present investigators in connection with a semi-elliptic formulation developed earlier and applied to the determination of separated internal flow.

TABLE 1: MEAN EXECUTION TIME (SEC.) FOR DIRICHLET PROBLEM USING AMDAHL 470 V/6 COMPUTER

GRID LEVEL	METHOD \ SR	1.57	2.87	8.00	10.14
N=9, M=8	SD	8.51×10^{-2}	4.65×10^{-1}	3.01×10^{0}	3.67×10^{0}
	BGE	3.16×10^{-3}	3.22×10^{-3}	3.28×10^{-3}	3.27×10^{-3}
N = M = 16	SD	7.78×10^{-1}	$1.79 \times 10^{+1}$	$1.58 \times 10^{+2}$	--
	BGE	2.28×10^{-2}	2.28×10^{-2}	2.27×10^{-2}	2.26×10^{-2}
N = M = 32	SD	6.19×10^{0}	$8.02 \times 10^{+2}$	--	--
	BGE	1.90×10^{-1}	1.89×10^{-1}	1.89×10^{-1}	1.87×10^{-1}

TABLE 2: MEAN EXECUTION TIME (SEC.) FOR BGE COEFFICIENT CALCULATIONS USING AMDAHL 470/V6 COMPUTER.

GRID LEVEL	N = 9, M = 8	N = M = 16	N = M = 32
Execution Time	8.14×10^{-3}	1.08×10^{-1}	1.74×10^{0}

TABLE 3: MEAN MAXIMUM ERROR FOR DIRICHLET PROBLEM USING DOUBLE-PRECISION ARITHMETIC

GRID LEVEL	METHOD \ SR	1.57	2.87	8.00	10.14
N=9, M=8	SD	2.66×10^{-7}	2.42×10^{-7}	3.72×10^{-7}	3.81×10^{-7}
	BGE	9.60×10^{-16}	7.20×10^{-16}	6.14×10^{-16}	1.05×10^{-15}
N = M = 16	SD	1.09×10^{-7}	1.27×10^{-7}	2.06×10^{-7}	--
	BGE	6.24×10^{-15}	3.53×10^{-15}	3.04×10^{-15}	2.18×10^{-15}
N = M = 32	SD	2.36×10^{-8}	6.42×10^{-8}	--	--
	BGE	5.80×10^{-14}	3.12×10^{-14}	1.16×10^{-14}	1.16×10^{-14}

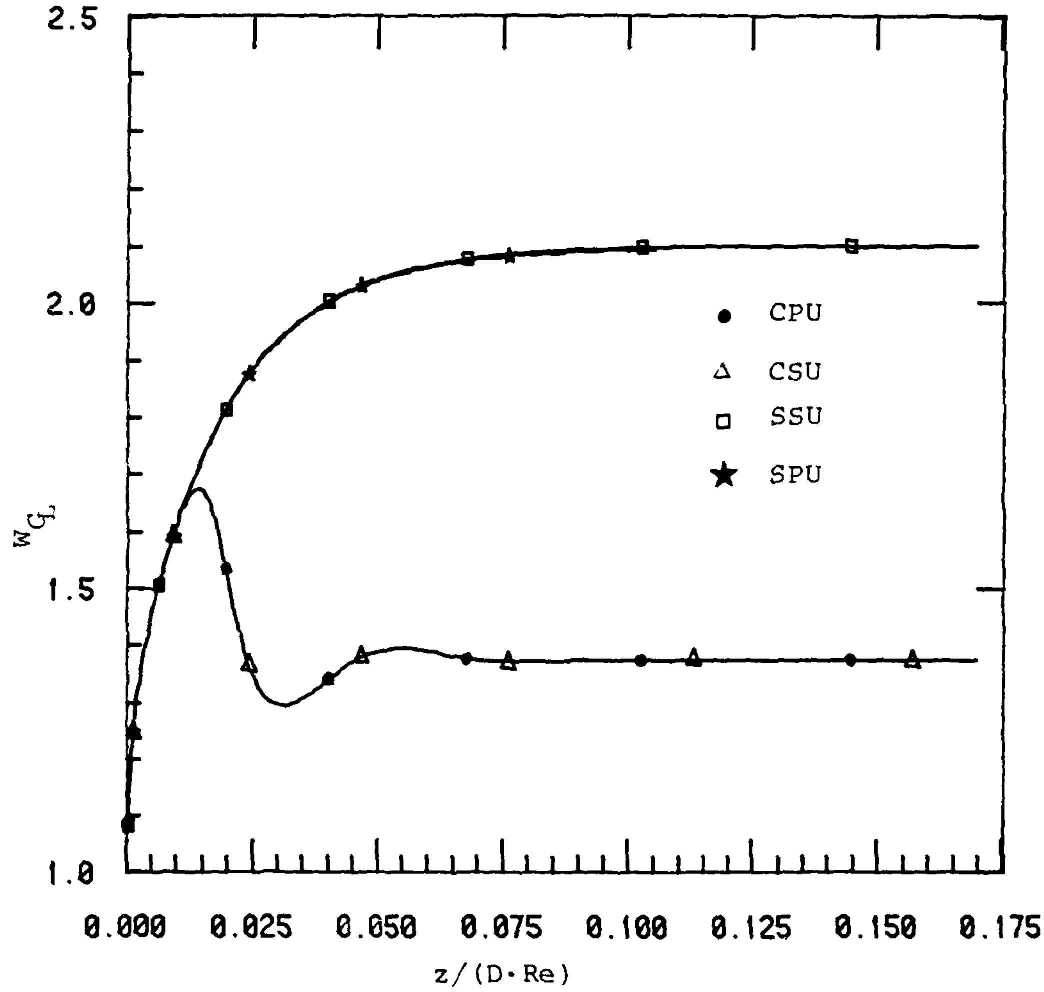

FIG. 1. STREAMWISE VARIATION OF CENTERLINE AXIAL VELOCITY.

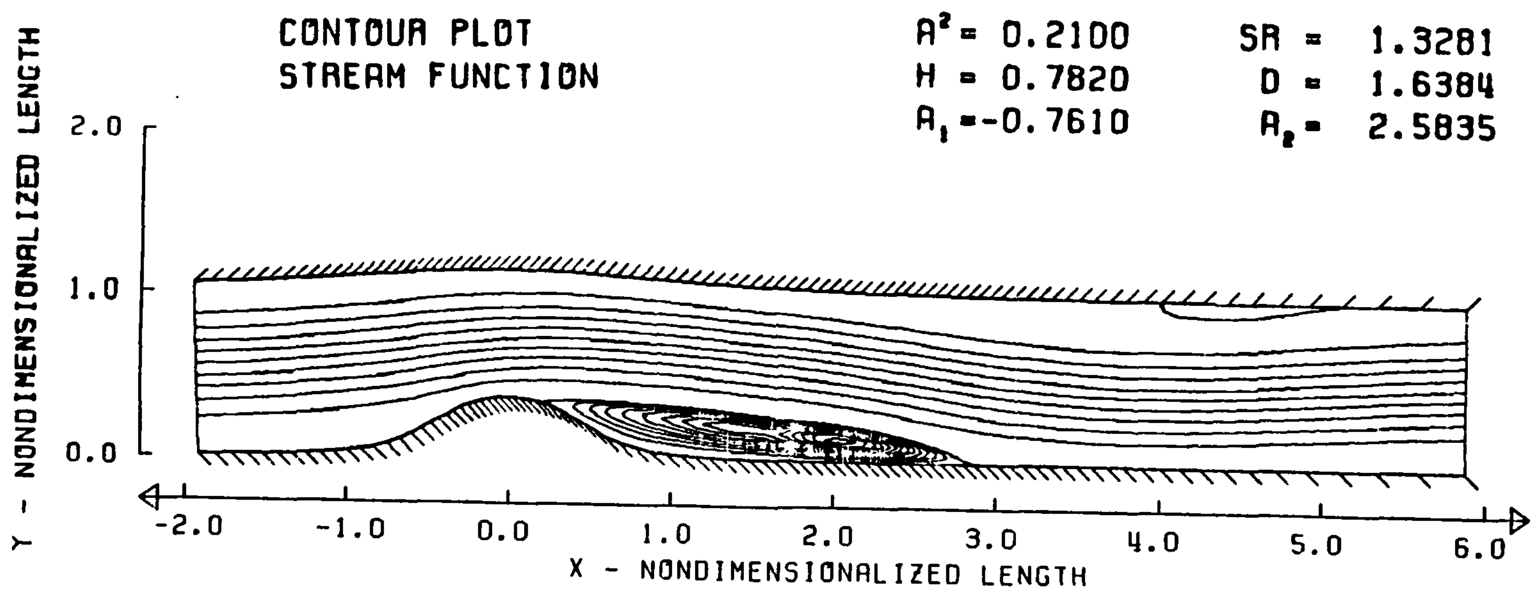

FIG. 2. TRANSIENT STREAM FUNCTION CONTOURS FOR Re = 1,000. $\Delta\psi$ = 0.002 WITHIN SEPARATION BUBBLE; $\Delta\psi$ = 0.1 OTHERWISE. (135,33) MESH. CHARACTERISTIC TIME = 8.00.

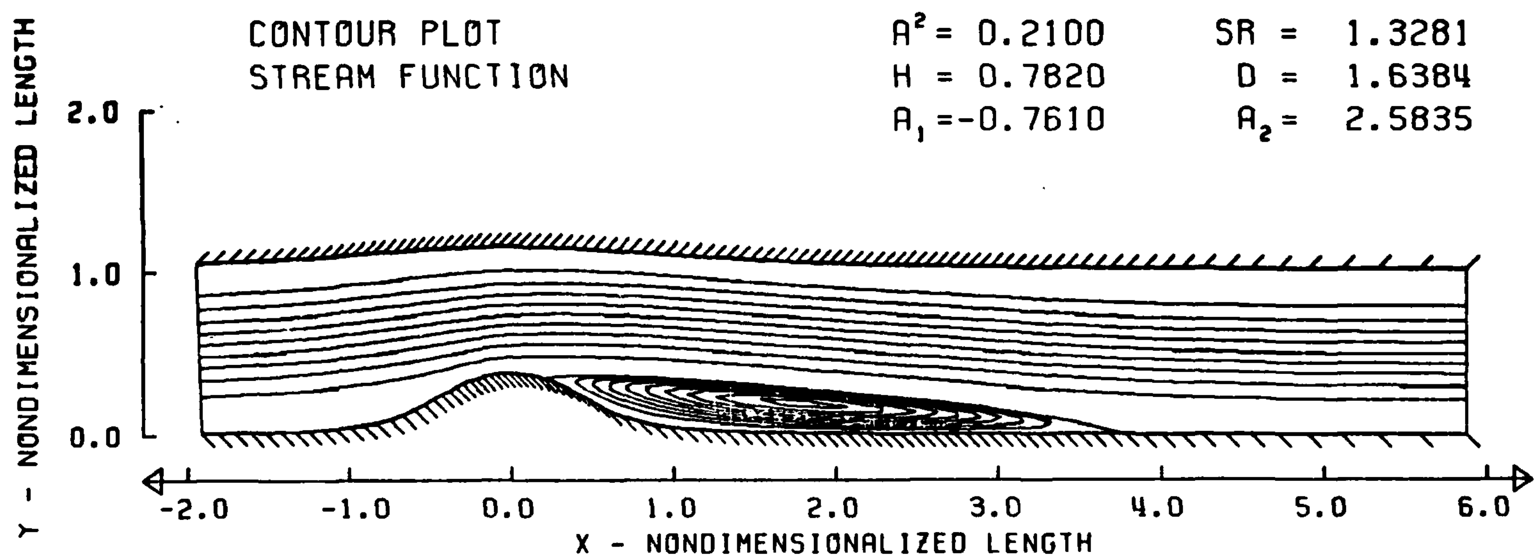

FIG. 3. STEADY-STATE STREAM FUNCTION CONTOURS FOR Re = 1,000. $\Delta\psi = 0.002$ WITHIN SEPARATION BUBBLE; $\Delta\psi = 0.1$ OTHERWISE. (135,33) MESH. CHARACTERISTIC TIME = 41.34.

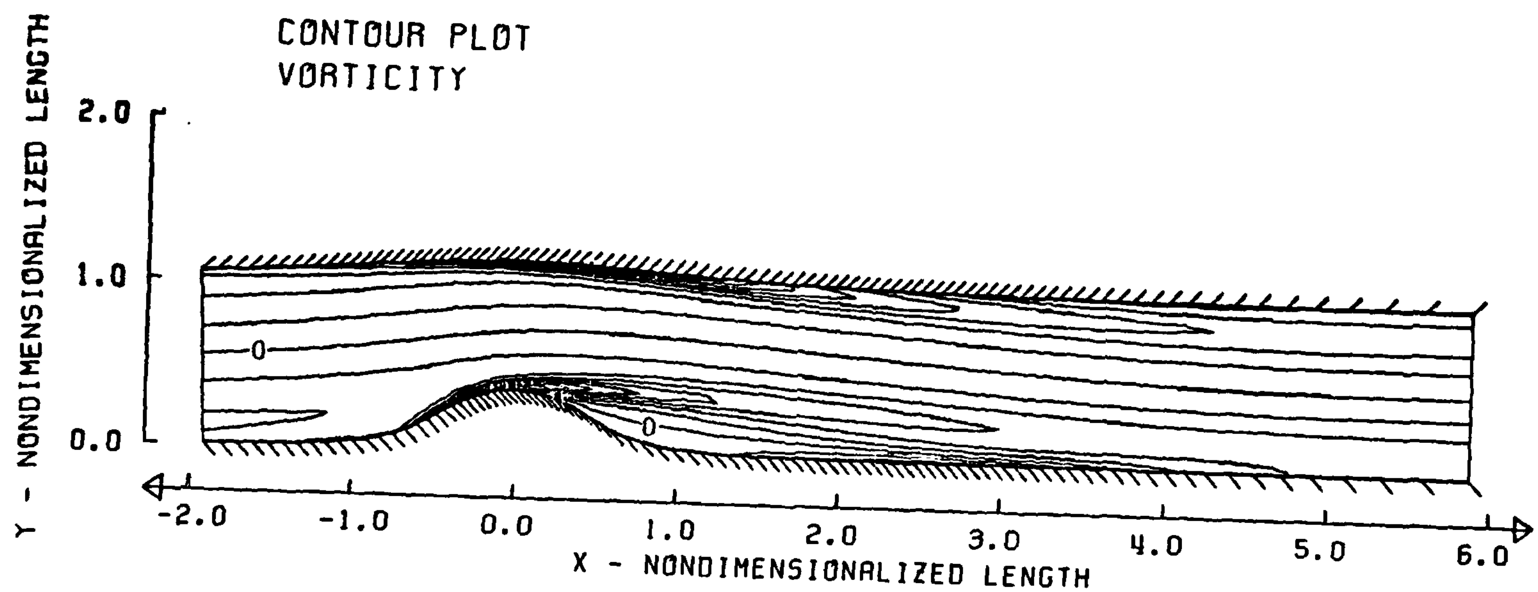

FIG. 4. STEADY-STATE VORTICITY CONTOURS FOR Re = 1,000, $\Delta\omega = 2.0$.

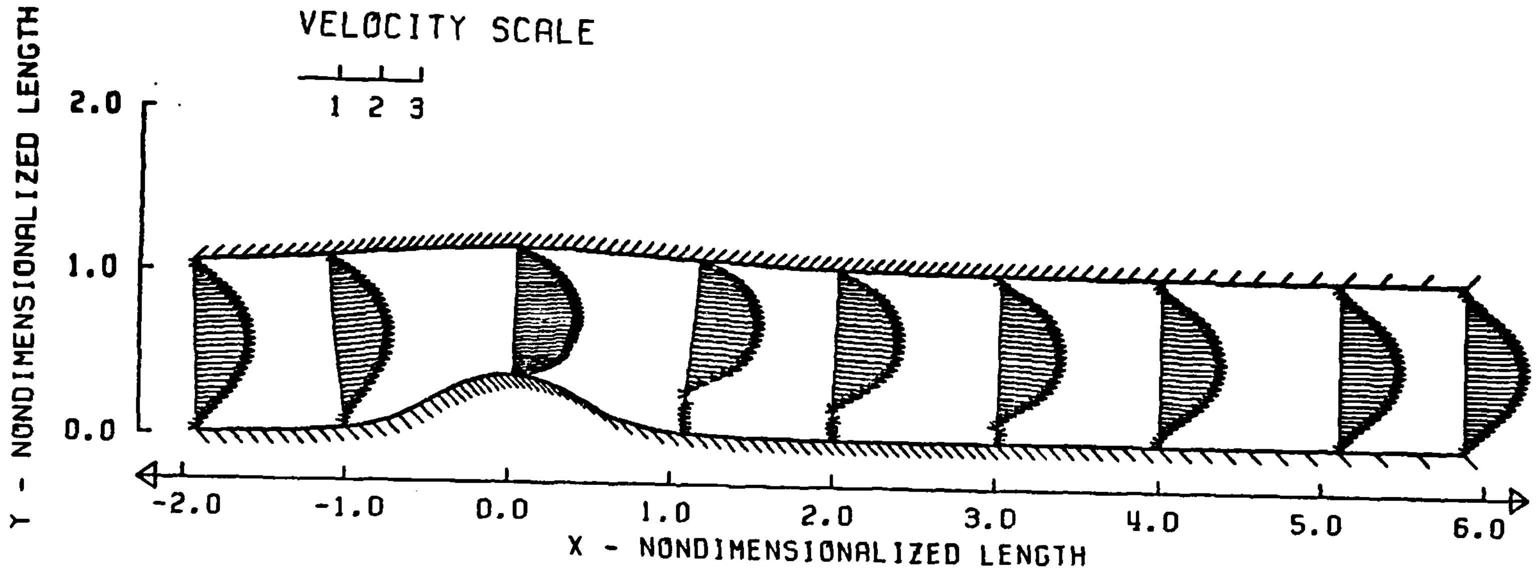

FIG. 5 . STEADY-STATE VELOCITY PROFILES FOR Re = 1,000.

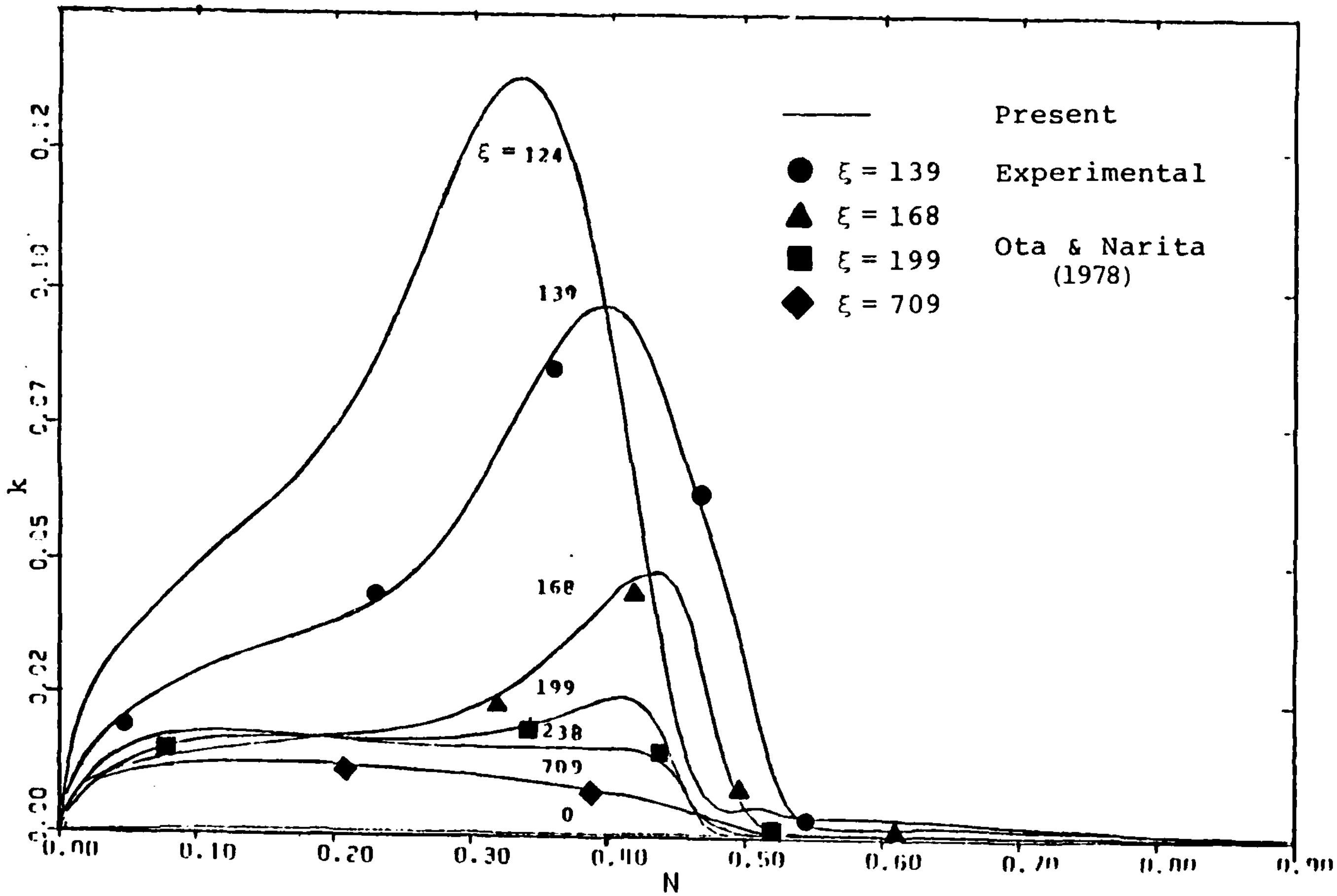

FIG. 6. COMPARISON BETWEEN PREDICTED AND EXPERIMENTAL VARIATION OF TURBULENT KINETIC ENERGY WITHIN REDUCED SEPARATION BUBBLE FOR BLUNT PLATES.

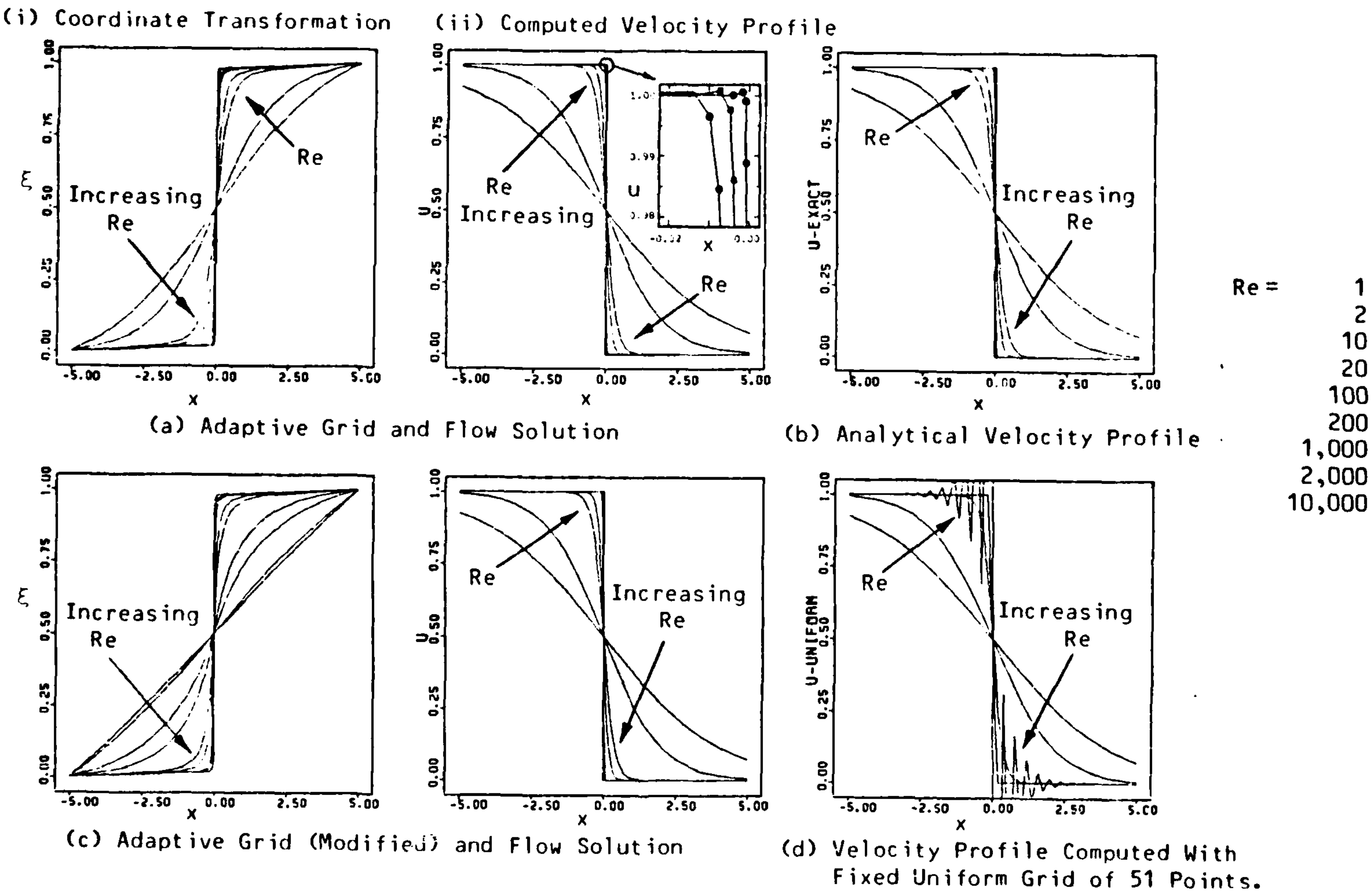

FIG. 7. SOLUTION OF 1-D VISCOUS BURGERS' EQUATION, WITH RE RANGING FROM 1 TO 10,000.

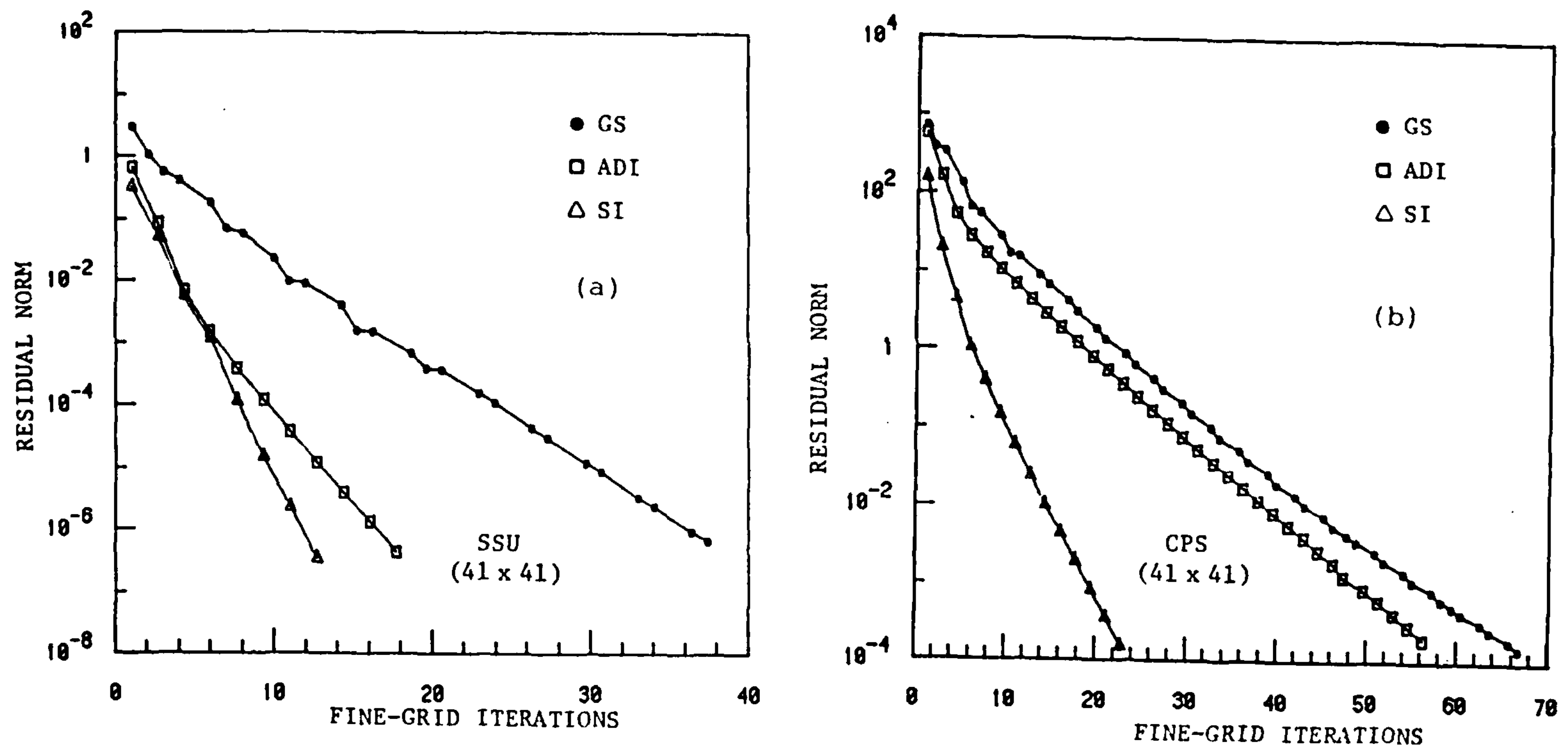

FIG. 8. COMPARATIVE STUDY FOR MULTI-GRID SCHEMES USING VARIOUS SMOOTHING OPERATORS - p-EQUATION, PLANE 3.

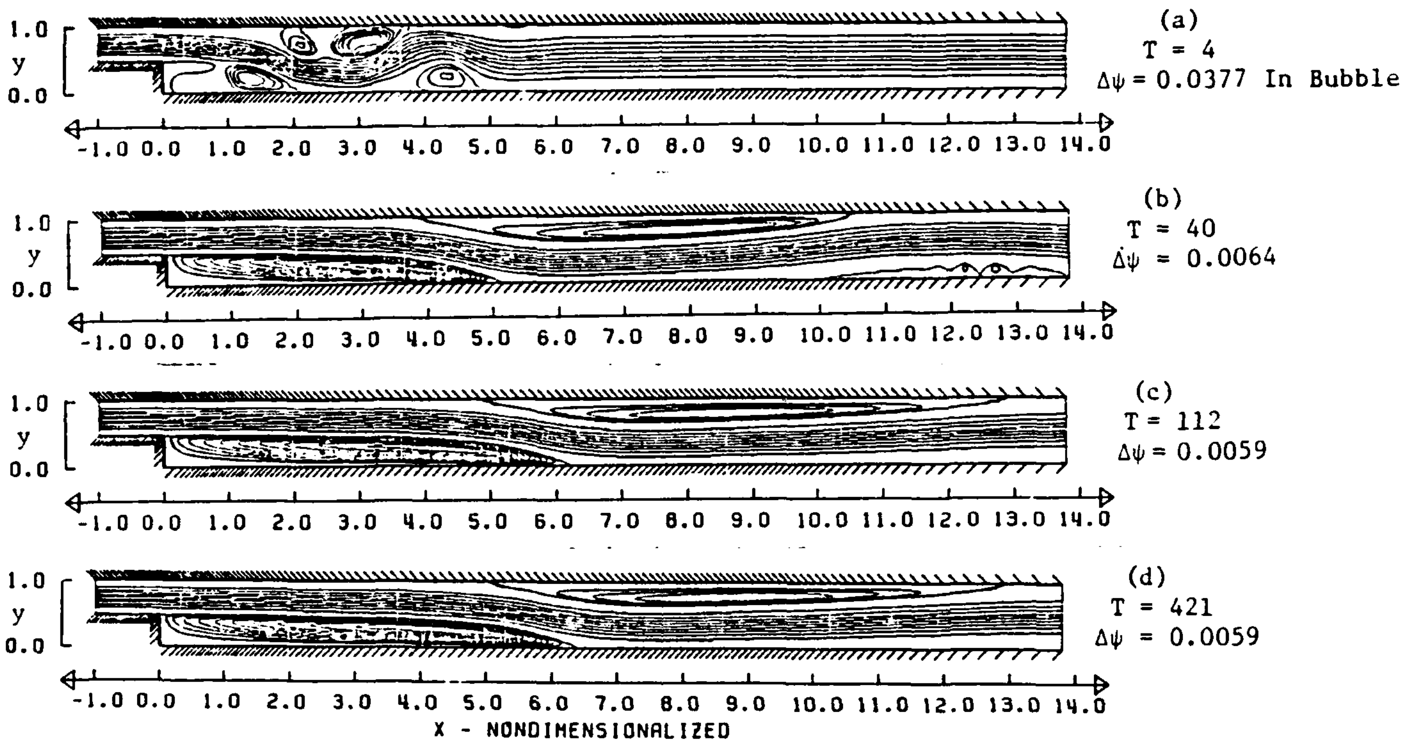

FIG. 9. TRANSIENT STREAM-FUNCTION CONTOURS FOR Re = 600; $\Delta\psi$ = 0.1.

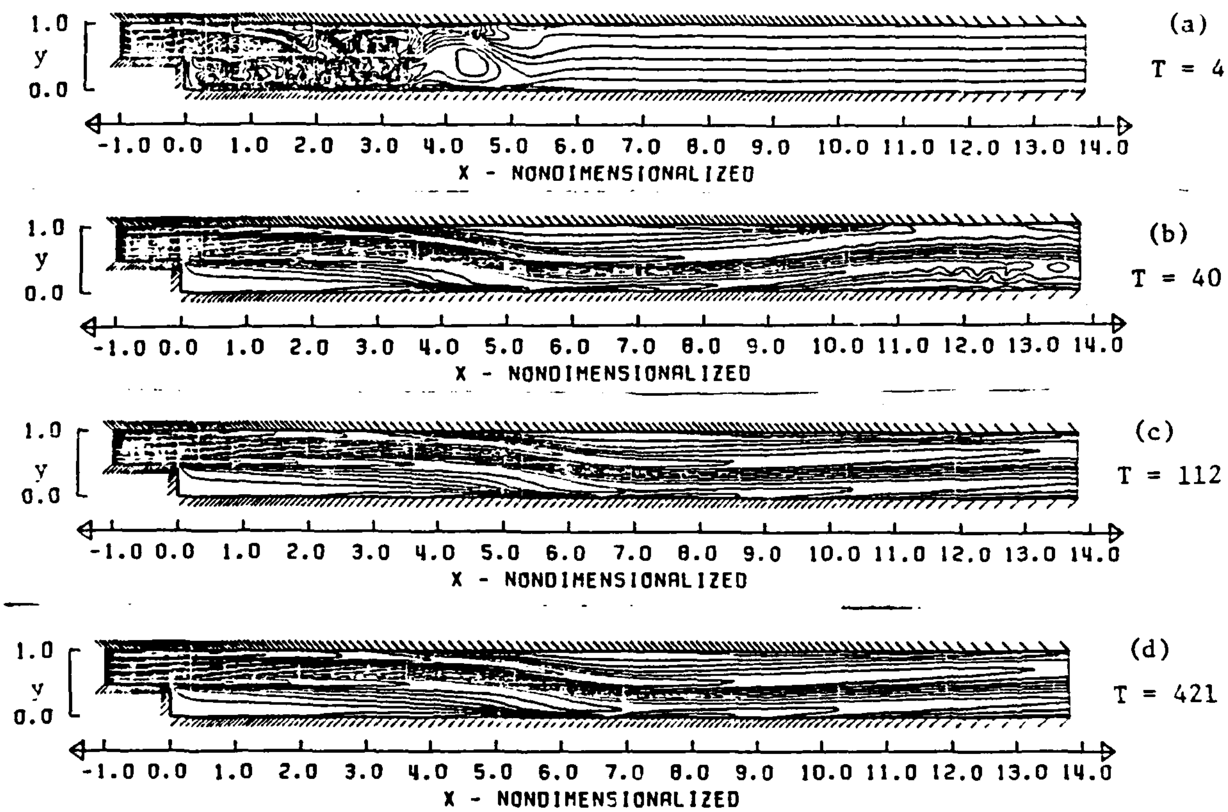

FIG. 10. TRANSIENT VORTICITY CONTOURS. Re = 600; $\Delta\omega = 2.0$.

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